

Pre-Calculus: Functions and Their Properties

Name: _____

Date: _____ Hour: _____

(Solving equations algebraically and graphically, matching graphs, tables, and equations, and finding the domain, range, VA, HA, etc.).

SCORE: _____/106
Percent Correct: _____%

Be sure to SHOW ALL WORK. Answer questions completely. Be sure to write answers in spaces provided. If work or answers are in another location, please make note of that. Short answer or multiple choice problems are worth 2 points each. Other problems will be worth 4 points each based on our department 4 point rubric. There are 106 points possible.

8/4	Correct, complete, with appropriate work or explanations.
6/3	Correct strategy, minor errors, appropriate work or explanations.
4/2	Starts with appropriate strategy, some understanding, some errors.
2/1	Attempted appropriate strategy, minimal understanding.
0	Little or no understanding evident - OR - no work shown.

I. Select the appropriate graph and equation for each table of values. (2 points per answer. 16 total)

1. Equation c Graph g

x	0	1	2	4
y	-1.67	-3.5	-9	13

3. Equation a Graph f

x	-3	-2	-1	0
y	1	1.414	1.732	2

2. Equation b Graph e

x	0	1	2	4
y	-1.67	-2.5	-5	5

4. Equation d Graph h

x	-2	-1	0	1
y	8	5	4	5

Equations:

a. $a(x) = \sqrt{x+4}$ <u>f</u>	b. $b(x) = \frac{5}{x-3}$ <u>e</u>	c. $c(x) = \frac{2x+5}{x-3}$ <u>g</u>	d. $d(x) = x^2 + 4$ <u>h</u>
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Graphs:

e.	f.	g.	h.

II. Solve each equation algebraically. (4 points each)

5. $3x^2 + 3x = 36$

$3x^2 + 3x - 36 = 0$
 $3(x^2 + x - 12) = 0$
 $3(x-3)(x+4) = 0$
 $x = 3 \quad x = -4$

6. $x^2 - 5x + 3 = 0$

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(3)}}{2}$
 $x = \frac{5 \pm \sqrt{25 - 12}}{2}$
 $x = \frac{5 \pm \sqrt{13}}{2}$
 $x = 4.3$
 $x = 2.70$

III. Complete the table below. Fill in every box. (1 point per sketch and 2 points per box. 78 points total.)

	Equations	Sketch the Graph	Domain	Range	Vertical Asymptote	Horizontal Asymptote	Max./Min. (x,y)	Hole Yes or no? Where?
7.	$y = \frac{2x}{x-5}$		$(-\infty, 5) \cup (5, \infty)$	$(-\infty, 2) \cup (2, \infty)$	$x=5$	$y=2$	none	none
8.	$y = \sqrt{2-x} - 3$		$(-\infty, 2]$	$[-3, \infty)$	none	none	$(2, -3)$ min	none
9.	$x = -3$		$x = -3$	$(-\infty, \infty)$	none	none	none	none
10.	$y = \frac{x-4}{x^2-x-12}$		$(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$	$(-\infty, 0) \cup (0, \infty)$	$x = -3$	$y = 0$	none	$x = 4$
11.	$y = x^2 + 7x + 12$		$(-\infty, \infty)$	$[-\frac{1}{4}, \infty)$	none	none	$(-\frac{7}{2}, -\frac{1}{4})$ min	none
12.	$y = - x-1 + 4$		$(-\infty, \infty)$	$(-\infty, 4]$	none	none	$(1, 4)$ max.	none

13. In what type of equation would we expect to find a HORIZONTAL ASYMPTOTE?
Explain and give an example to support your answer. (4 points)

Rational functions. $f(x) = \frac{ax^n}{bx^m}$

- i. $n < m$; $y = 0$
- ii. $n = m$; $y = a/b$
- iii. $n > m$; $y = \text{none}$ (or slant)

14. **EXTRA CREDIT:** Give an example of a function that fits the criteria below. There may be more than one possible solution. (Pick ONLY 2. 2 points each)

1. Domain: $(-\infty, \infty)$
Range: $(-\infty, -3)$ $-|x| - 3$

2. Domain: $(-3, \infty)$
Range: $(2, \infty)$ $\sqrt{x+3} + 2$

3. Vertical Asymptote: $x = -1$
Horizontal Asymptote: $y = 0$
Hole: $x = 4$
 $\frac{x-4}{(x-4)(x+1)}$

1.3-1.5

~~1.3-1.5~~

Key

Pre-Calculus: Graph Transformations and Combining Functions.

(Transforming images and parent functions as well as combining functions using the five different properties.)

Name: _____

Date: _____ Hour: _____

SCORE: ____ / 64

Percent Correct: ____ %

Be sure to SHOW ALL WORK.

Answer questions completely. Be sure to write answers in spaces provided. If work or answers are in another location, please make note of that. There are 64 points possible.

16/8/4	Correct, complete, with appropriate work or explanations.
12/6/3	Correct strategy, minor errors, appropriate work or explanations.
8/4/2	Starts with appropriate strategy, some understanding, some errors.
4/2/1	Attempted appropriate strategy, minimal understanding.
0	Little or no understanding evident - OR - no work shown.

1. Find $(f + g)(x)$ and $(f - g)(x)$ given that $f(x) = 3x + 2$ and $g(x) = x^2 - 7$. (4 points)

$$= 3x + 2 + x^2 - 7$$

$$= x^2 + 3x - 5$$

$$3x + 2 - (x^2 - 7)$$

$$3x + 2 - x^2 + 7$$

a) $(f + g)(x) = x^2 + 3x - 5$

b) $(f - g)(x) = -x^2 + 3x + 9$

2. Find $(f * g)(-2)$ and $(\frac{f}{g})(3)$ given that $f(x) = \sqrt{x + 6}$ and $g(x) = x^2 - 1$. (4 points)

$$= (\sqrt{x + 6})(x^2 - 1)$$

$$= (\sqrt{-2 + 6})((-2)^2 - 1)$$

$$= (2)(3)$$

$$\frac{\sqrt{x + 6}}{x^2 - 1} = \frac{3}{8}$$

a) $(f * g)(-2) = 6$

b) $(\frac{f}{g})(3) = \frac{3}{8}$

3. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ with $f(x) = 3x + 5$ and $g(x) = x^2 - 4$ (4 points each)

A. $(f \circ g)(x)$

$$\begin{aligned} f(g(x)) &= f(x^2 - 4) \\ &= 3(x^2 - 4) + 5 \\ &= 3x^2 - 12 + 5 \\ &= 3x^2 - 7 \end{aligned}$$

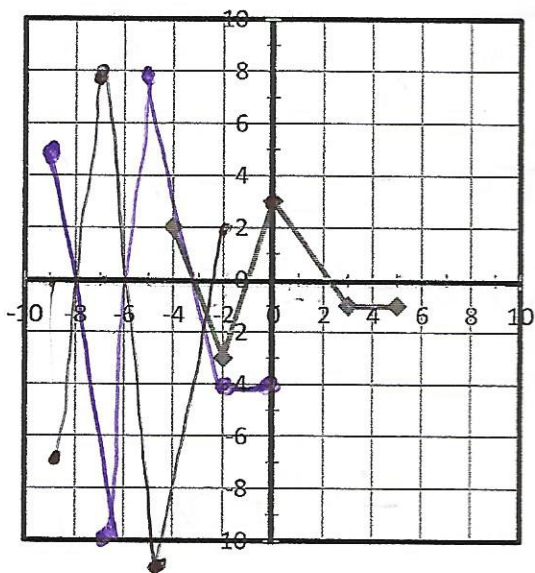
a) $(f \circ g)(x) = \underline{3x^2 - 7}$

B. $(g \circ f)(x)$

$$\begin{aligned} g(f(x)) &= g(3x + 5) \\ &= (3x + 5)^2 - 4 \\ &= 9x^2 + 30x + 25 - 4 \\ &= 9x^2 + 30x + 21 \end{aligned}$$

b) $(g \circ f)(x) = \underline{9x^2 + 30x + 21}$

4. Perform the appropriate graph transformations and draw the new graph. (16 points each)



HSRV

$$y = 3f(-x + 5) - 1$$

x	y
-4	2
-2	-3
0	3
3	-1
5	-1

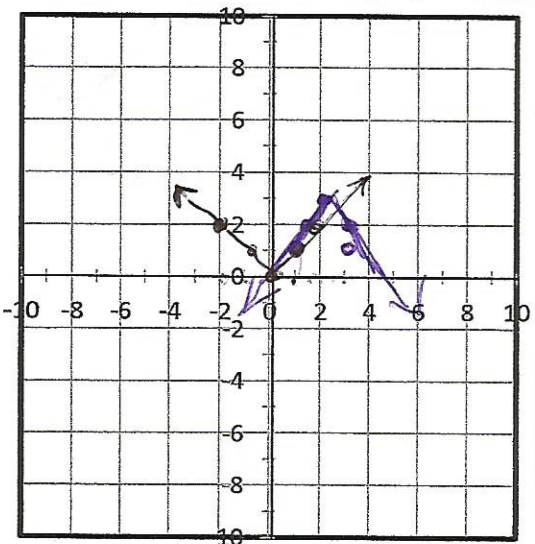
x	y
-9	2
-7	-3
-5	3
-2	-1
0	-1

x	y
-9	6
-7	-9
-5	9
-2	-3
0	-3

x	y
+9	+6
+7	-9
+5	+9
+2	-3
0	3

x	y
+9	5
+7	-8
+5	8
+2	8
0	8

5. Perform the appropriate graph transformations and draw the new graph. (16 points each)



HSRV

$$f(x) = -|2x - 4| + 3$$

4

Parent Function: $y = |x|$

reflection over x axis

x	y
-2	2
-1	1
0	0
1	1
2	2

x	y
2	2
3	1
4	0
5	1
6	2

x	y
1	2
3/2	1
2	0
5/2	1
3	2

x	y
1	-2
3/2	-1
2	0
5/2	-1
3	-2

x	y
1	1
3/2	2
2	3
5/2	2
3	1

6. Find the inverse of the function: $f(x) = \frac{3x-5}{x+4}$. (8 points each)

$$y = \frac{3x-5}{x+4}$$

$$x = \frac{3y-5}{y+4}$$

$$\begin{aligned} x(y+4) &= 3y-5 & f^{-1} &= \frac{4x+5}{3-x} \\ xy+4x &= 3y-5 \end{aligned}$$

$$4x+5 = 3y - xy$$

$$\frac{4x+5}{3-x} = \frac{y(3-x)}{3-x}$$

$$f^{-1}(x) = \frac{4x+5}{3-x}$$

7. State the transformations occurring in each function below. (4 Points each)

A. $f(x) = 5\sqrt{-x+3} - 2$

- H = horizontal shift to left 3 units
- S = vertical stretch by factor of 5
- R = reflection over y axis
- V = vertical shift down 2

B. $f(x) = -\left(\frac{1}{2}x - 4\right)^3 + 7$

- H = horizontal shift to right 4
- S = hor. stretched by a factor of 2
- R = reflection over x axis
- V = vertical shift up 7 units

8. **Extra Credit:** What are the ten parent functions? (½ point each)

1. $y = x$

2. $y = x^2$

3. $y = x^3$

4. $y = |x|$

5. $y = \sqrt{x}$

6. $y = \frac{1}{x}$

7. $y = \cos(x)$

8. $y = \sin(x)$

9. $y = \ln(x)$

10. $y = e^x$

9. What grade do you deserve? _____

10. What grade do you want? _____

11. What can Mrs. Valentine or Ms. Orban do to help you do better?

Key

Pre-Calculus: 2.1 – 2.4
Polynomial, Power and Monomial
Functions, Graph Behavior and
Division

Name: _____

Date: _____ Hour: _____

SCORE: ____ / 72

Percent Correct: ____ %

Be sure to SHOW ALL WORK. Answer questions completely. Be sure to write answers in spaces provided. If work or answers are in another location, please make note of that.

There are 72 points possible.

16/8/4	Correct, complete, with appropriate work or explanations.
12/6/3	Correct strategy, minor errors, appropriate work or explanations.
8/4/2	Starts with appropriate strategy, some understanding, some errors.
4/2/1	Attempted appropriate strategy, minimal understanding.
0	Little or no understanding evident – OR – no work shown.

1. Using the Rational Zero Theorem, find all of the zeros for the polynomial function. (12 points)

$$f(x) = x^4 + 9x^3 + 14x^2 - 54x - 120$$

$$\begin{array}{r|l} x & y \\ -4 & 0 \\ -5 & 0 \end{array}$$

$$\begin{array}{r|rrrrrr} -4 & 1 & 9 & 14 & -54 & -120 \\ & & -4 & -20 & 24 & 120 \\ \hline -5 & 1 & 5 & -6 & -30 & 0 \\ & & -5 & 0 & 30 & \\ \hline & 1 & 0 & -6 & 0 & \end{array}$$

$$\begin{aligned} x^2 - 6 &= 0 \\ \sqrt{x^2} &= \sqrt{6} \\ x &= \pm\sqrt{6} \end{aligned}$$

$$x = \frac{-0 \pm \sqrt{0 - 4(1)(-6)}}{2}$$

Zeros: -4, -5, ±√6

2. State the end behavior for the following functions: (6 points)

i. $f(x) = -5x^3 + 4x^2 - 8$

∞ and -∞

ii. $f(x) = -3x^6 - x^4 + 7x^2 + 2$

-∞ and -∞

iii. $f(x) = 2x^5 - 5x + 9$

-∞ and ∞

3. Find the vertex and axis of symmetry for the following quadratic function. (4 points)

$$f(x) = -2x^2 - 12x + 4$$

$$x = \frac{-b}{2a} = \frac{-(-12)}{2(-2)} = -3$$

$$\begin{aligned} f(-3) &= -2(-3)^2 - 12(-3) + 4 \\ &= -18 + 36 + 4 \\ &= 18 + 4 \end{aligned}$$

Vertex: (-3, 22)
Axis of Symmetry: x = -3

4. Divide $f(x) = -4x^4 + x^3 + 2x^2 + 3x - 1$ by $d(x) = x - 1$. (4 points)

$$\begin{array}{r} \overline{) -4 \ 1 \ 2 \ 3 \ -1} \\ \underline{-4 \ -3 \ -1 \ 2} \\ -4 \ -3 \ -1 \ 2 \ \underline{+1} \end{array}$$

Fraction
Factor Form:

$$\frac{-4x^3 - 3x^2 - x + 2 + \frac{1}{x-1}}{x-1}$$

5. Determine whether the following are polynomial, power or monomial functions. If so, state by underlining the correct term and filling in the blank appropriately. (8 points)

A. $f(x) = 4x^3$

Circle all that apply:

Polynomial Power Monomial

Degree/Power: 3

Leading Coefficient/C.O.V.: 4

B. $f(x) = -3x^5 + 2x^3 - 5$

Circle all that apply:

Polynomial Power Monomial

Degree/Power: 5

Leading Coefficient/C.O.V.: -3

6. Find the quadratic equation that has a vertex of $(-4, 13)$ and point $(-6, 1)$. (4 points)

$$\begin{aligned} y &= a(x-h)^2 + k \\ 1 &= (-6 - (-4))^2 + 13 \\ 1 &= a(-2)^2 + 13 \\ 1 &= 4a + 13 \\ \frac{-12}{4} &= \frac{4a}{4} \quad a = -3 \end{aligned}$$

Final Equation: $y = -3(x+4)^2 + 13$

7. State the degree and zeros of the polynomial function. State the multiplicity of each zero and what the behavior of the graph is at that zero (crosses/kisses). (10 points)

Degree: 8

$$f(x) = (x + 5)^4(x - 2)(x - 6)^3$$

Zeros	Multiplicity	Crosses/Kisses
$x = -5$	4	Kisses
$x = 2$	1	Crosses
$x = 6$	3	Crosses

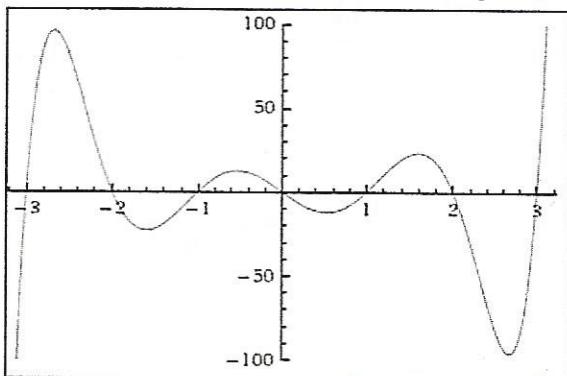
8. Write the statements below as a power function equation. (4 points)

m varies directly with the fourth root of t .	$m = k\sqrt[4]{t}$
g is inversely proportional to the cube of f .	$g = k/f^3$

9. Write a sentence that expresses the relationship in the power formula, using the language of variation or proportion. (4 points)

$y = -5x^{-3}$	Y varies inversely to the cube of x with a constant variation of -5
$V = \frac{4}{3}\pi r^3$ ($V = \text{Volume}$ and $r = \text{radius}$)	The volume varies directly with the cube of the radius with a constant variation of $\frac{4}{3}\pi$

10. Given the graph, state the following: (4 points)



- i. Number of Zeros: 7
- ii. Number of Extrema: 6

11. Using long division, divide $f(x) = 3x^4 + 2x^3 + 10x^2 + 4x - 5$ by $d(x) = x^2 + 2$. (8 points)

$$\begin{array}{r}
 3x^2 + 2x + 4 \\
 \hline
 x^2 + 0x + 2 \overline{) 3x^4 + 2x^3 + 10x^2 + 4x - 5} \\
 \underline{3x^4 + 0x^3 + 6x^2} \\
 2x^3 + 4x^2 + 4x \\
 \underline{2x^3 + 0x^2 + 4x} \\
 4x^2 + 0x - 5 \\
 \underline{4x^2 + 0x + 8} \\
 -13
 \end{array}$$

Polynomial Form:

$$(3x^2 + 2x + 4)(x^2 + 2) - 13$$

12. Write the equation for the linear equation with the points $(-3, 5)$ and $(-4, 8)$. (4 points)

$$\begin{aligned}
 y &= mx + b \\
 m &= \frac{8 - 5}{-4 - (-3)} = \frac{3}{-1} = -3 \\
 8 &= -3(-4) + b \\
 8 &= 12 + b \\
 -12 &= -12 \\
 -4 &= b
 \end{aligned}$$

Final Equation: $y = -3x - 4$

Pre-Calculus: 2.5 – 2.6
Complex Numbers and the Fundamental Theorem of Algebra

Name: _____

Date: _____ Hour: _____

SCORE: _____ / 88

Percent Correct: _____ %

Be sure to SHOW ALL WORK. Answer questions completely. Be sure to write answers in spaces provided. If work or answers are in another location, please make note of that.

There are 88 points possible.

16/8/4	Correct, complete, with appropriate work or explanations.
12/6/3	Correct strategy, minor errors, appropriate work or explanations.
8/4/2	Starts with appropriate strategy, some understanding, some errors.
4/2/1	Attempted appropriate strategy, minimal understanding.
0	Little or no understanding evident – OR – no work shown.

1. Perform the indicated operation and write the result in standard form. (16 points)

<p>A. $(3 - 2i) + (-2 + 5i)$</p> <p>$3 + -2 - 2i + 5i$</p> <p>Solution: <u>$1 + 3i$</u></p>	<p>B. $(5 - 7i) - (3 - 2i)$</p> <p>$5 - 3 - 7i - (+2i)$</p> <p>Solution: <u>$2 - 5i$</u></p>	<p>C. $(1 + 2i)(3 - 2i)$</p> <p>$1(3 - 2i) + 2i(3 - 2i)$</p> <p>$3 - 2i + 6i - 4i^2$</p> <p>Solution: <u>$7 + 4i$</u></p>	<p>D. i^{123}</p> <p>$\frac{123}{4} = 30.75$</p> <p>$(i^4)^{30} \cdot i^3 = 1 \cdot i^3$</p> <p>Solution: <u>$-i$</u></p>
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2. Write the expression in bi where b is a real number. (4 points)

<p>A. $\sqrt{-81}$</p> <p>Solution: <u>$9i$</u></p>	<p>B. $\sqrt{-625}$</p> <p>Solution: <u>$25i$</u></p>
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3. Write the following expressions in standard form. (12 points)

<p>A. $\frac{(2i)(3-4i)}{3+i}$</p> <p>$(2i)(3-4i) = 6i - 8i^2 = 8 + 6i$</p> <p>$\frac{8 + 6i}{3+i} \cdot \frac{3-i}{3-i} = \frac{(8 + 6i)(3-i)}{9 - i^2} = \frac{24 - 8i + 18i - 6i^2}{9 + 1} = \frac{30 + 10i}{10} = 3 + i$</p> <p>Solution: <u>$3 + i$</u></p>	<p>A. $(1 + 3i)^3$</p> <p>$(1 + 3i)(1 + 3i) = 1 + 6i + 9i^2 = -8 + 6i$</p> <p>$(-8 + 6i)(1 + 3i) = -8 - 24i + 6i + 18i^2 = -26 - 18i$</p> <p>Solution: <u>$-26 - 18i$</u></p>
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4. Find the product of the complex number and its conjugate given that $f(x) = 2 - 9i$. (4 points)

$(2 - 9i)(2 + 9i)$

$4 + 18i - 18i - 81i^2$

$4 + 81$

Solution: 85

5. Write in standard form a polynomial function that would have with real coefficients and zeros: 5, -1 and 4 + i. (Multiply out) (12 points)

$x=5$ $(x-5)(x+1)(x-4-i)(x-4+i)$
 $x=-1$ x^2-4x-5 $x^2-4x+ix-4x+16-4i-ix+4i-x^2$
 $x=4+i$ $x^2-8x+17$
 $x=4-i$

$(x^2-4x-5)(x^2-8x+17)$
 $x^2(x^2-8x+17) - 4(x^2-8x+17) - 5(x^2-8x+17)$
 $x^4 - 8x^3 + 17x^2 - 4x^3 + 32x^2 - 68x - 5x^3 + 40x - 85$

Standard Form: $x^4 - 12x^3 + 44x^2 - 28x - 85$ ✓

6. Write in standard form a polynomial function that would have with real coefficients and zeros with their multiplicities: (Multiply out) (12 points)

Zero	Multiplicity
-4	2
3i	1

$(x+4)(x+4)(x-3i)(x+3i)$
 $x^2+8x+16$ $x^2+3ix-3ix-9i^2$
 x^2+9

$(x^2+8x+16)(x^2+9)$
 $x^2(x^2+9) + 8x(x^2+9) + 16(x^2+9)$
 $x^4 + 9x^2 + 8x^3 + 72x + 16x^2 + 144$

Standard Form: $x^4 + 8x^3 + 25x^2 + 72x + 144$ ✓

7. Find all of the zeros and write a linear factorization of the function. (12 points)

$f(x) = x^4 + 3x^3 - x^2 + 2x - 40$

$x=2$ $2 \mid 1 \quad 3 \quad -1 \quad 2 \quad -40$
 $x=-4$ $\downarrow \quad 2 \quad 10 \quad 18 \quad 40$
 $-4 \mid 1 \quad 5 \quad 9 \quad 20$
 $\downarrow \quad -4 \quad -4 \quad 20$
 $1 \quad 1 \quad 5 \quad \emptyset$

x^2+x+5
 $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1^2-4(1)(5)}}{2(1)}$
 $= \frac{-1 \pm \sqrt{-19}}{2}$
 $= \frac{-1 \pm \frac{\sqrt{19}}{2}i}{2}$

Linear Factorization: $(x-2)(x+4)(x + \frac{-1 + \frac{\sqrt{19}}{2}i}{2})(x + \frac{-1 - \frac{\sqrt{19}}{2}i}{2})$

3. Given the zero $5 + 3i$, find all of the zeros and write a linear factorization of the function. (8 points)

$$f(x) = x^3 - 6x^2 - 6x + 136.$$

$5+3i$		-6	-6	136	$x+4=0$ $x=4$
	↓	$5+3i$	$-14+2i$	-136	
$5-3i$		$-1+3i$	$-26+12i$	\emptyset	
	↓	$5-3i$	$20-12i$	\emptyset	
	↓	4	\emptyset	\emptyset	

Linear Factorization: $(x-5-3i)(x-5+3i)(x+4)$

Conceptual Questions: (8 Points)

1. Is it possible to get a 5th degree polynomial with real coefficients and zeros of $3 - 5i$ and $4 - i$? Explain.

No, each of these zeros have a conjugate so you only get 4 zeros. Thus a 4th degree polynomial.

2. Up to what exponent of i do you need to know in order to solve i^n when n is a positive constant? Why? Provide an example.

i^4 because after that the pattern repeats.

3. If you have an x^4 polynomial, what do you do if you can only find one zero in the table?

Do p/q because the zero is a fraction.

4. When given a polynomial function, how can we determine the number of zeros without actually solving for them? Explain.

The number of zeros depends on the highest degree of the function.

Pre-Calculus: 2.7 - 2.8

Rational Functions and Solving Inequalities in One Variable

Name: _____

Date: _____ Hour: _____

SCORE: 80 / 80 **92**

Percent Correct: _____%

Be sure to SHOW ALL WORK. Answer questions completely. Be sure to write answers in spaces provided. If work or answers are in another location, please make note of that. There are 80 points possible.

16/8/4	Correct, complete, with appropriate work or explanations.
12/6/3	Correct strategy, minor errors, appropriate work or explanations.
8/4/2	Starts with appropriate strategy, some understanding, some errors.
4/2/1	Attempted appropriate strategy, minimal understanding.
0	Little or no understanding evident - OR - no work shown.

1. Describe how the graphs of the given function can be obtained by transforming the graph of the rational function $f(x) = \frac{1}{x}$. (8 points)

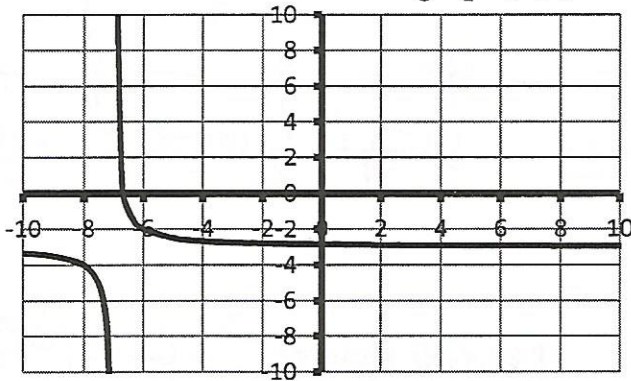
$$f(x) = \frac{-8x + 3}{x - 2} \quad y = 1$$

$x-2 \overline{) -8x+3}$
 $\underline{-(8x+16)}$
 $\quad -13$

$s(x) = -8 - \frac{13}{x-2}$

H - right 2 units
 S - stretch by a factor of 8
 R - over the x axis
 V - down 8 units

2. Evaluate the limit based on the graph $f(x)$ shown. (8 points)



$\lim_{x \rightarrow -7^+} f(x) = \infty$
 $\lim_{x \rightarrow -7^-} f(x) = -\infty$
 $\lim_{x \rightarrow \infty} f(x) = -3$
 $\lim_{x \rightarrow -\infty} f(x) = -3$

3. Solve the following equation algebraically. (12 points)

$$\frac{1}{x-4} + \frac{x}{x-2} = \frac{2}{x^2 - 6x + 8}$$

$$\frac{1}{x-4} + \frac{x}{x-2} = \frac{2}{(x-4)(x-2)}$$

$$x-2 + x(x-4) = 2$$

$$x-2 + x^2 - 4x = 2$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

~~x=4~~ or x=-1

Solution: X = -1

4. For the given functions, find the indicated values. (12 points)

A. $f(x) = \frac{4x+3}{x^2-16}$

B. $f(x) = \frac{x^2-5x-36}{x+3}$

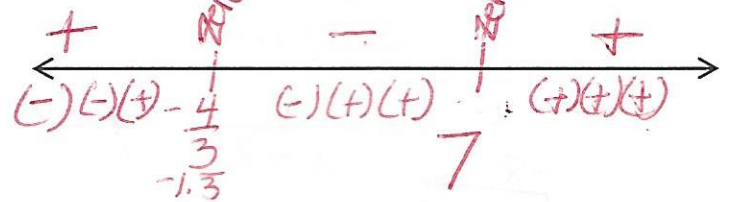
$0 = \frac{4x+3}{x^2-16}$
 $0 = 4x+3$
 $-3 = 4x$
 $-\frac{3}{4} = x$
 X-Intercepts: $-\frac{3}{4}$ Y-Intercept: $-\frac{3}{16}$

$x-8$
 $x+3 \overline{) x^2-5x-36}$
 $-x^2+3x$
 $-8x-36$
 $-8x-24$
 -12
 VA: $x=-3$ HA: none Slant A.: $y=x-8$

For the following problems you are **REQUIRED** to make a sign chart for full credit.

5. Determine the x values that cause the polynomial functions to be (a) zero, (b) positive, and (c) negative. (8 points)

$f(x) = (x-7)(3x+4)(x^2+6)$
 $x=7$ $x=-\frac{4}{3}$



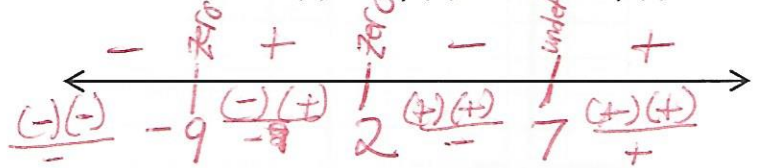
- (a): $x=7$ $x=-\frac{4}{3}$ (b): $(-\infty, -\frac{4}{3})$ $x < -\frac{4}{3}$ $(-\frac{4}{3}, 7)$ $x > 7$ (c): $-\frac{4}{3} < x < 7$

6. Determine the x values that cause the polynomial function to be (a) zero, (b) undefined, (c) positive, and (d) negative. (12 points)

A. $f(x) = \frac{(3x-6)(x+9)}{x-7}$

$3x-6=0$
 $x=2$

$x=-9$

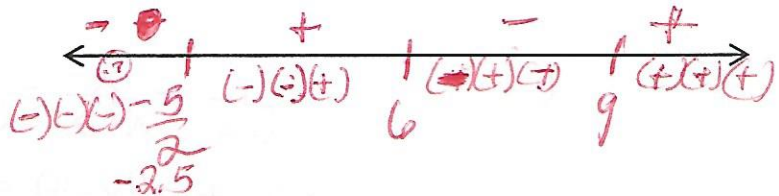


- (a): $x=2, x=-9$ (b): $x=7$ (c): $(-\infty, -9)$ $x < -9$ $(-9, 2)$ $-9 < x < 2$ $(2, 7)$ $2 < x < 7$ (d): $(7, \infty)$ $x > 7$

7. Solve the polynomial inequality. (12 points)

A. $2x^3 - 25x^2 + 33x + 270 > 0$

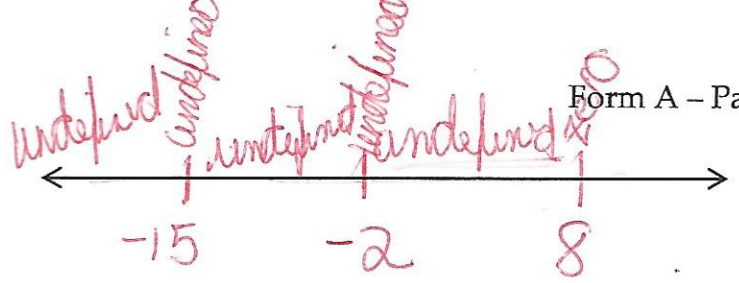
$9 \overline{) 2 \ -25 \ 33 \ 270}$
 $\underline{18 \ -63 \ -270}$
 $6 \overline{) 2 \ -7 \ -30}$
 $\underline{12 \ 30}$
 $2 \ 5$
 $2x+5=0$
 $2x=-5$
 $x=-\frac{5}{2}$



$f(x) = (x-9)(x-6)(2x+5)$
 Solution: $(\frac{5}{2}, 6)$ $(9, \infty)$

B. $\frac{\sqrt{x-8}}{x^2+17x+30} \leq 0$

$\frac{\sqrt{x-8}}{(x+15)(x+2)}$



12

[8]

Solution: ~~no solution~~

8. Answer the following questions in complete sentences. (8 points)

A. When is a polynomial function undefined? (HINT: There are two cases).

I. when the denominator is equal to zero

4

II. when the denominator has a radical, the function is undefined where the radicand is less than zero

B. How do we find horizontal asymptotes? (Give an example of each!)

4

1) $f(x) = \frac{x-5}{x^2+2}$ degree of denominator higher
HA $y = 0$

2) $f(x) = \frac{3x-2}{5x+3}$ degrees of denominator and numerator equal, then divide leading coefficients
 $y = \frac{3}{5}$

3) $f(x) = \frac{x^2+2}{x-5}$ degree of numerator higher than denominator it has a slant asymptote, must perform division

$$\begin{array}{r} x+5 \\ x-5 \overline{) x^2+0x+2} \\ \underline{x^2-5x} \\ 5x+2 \end{array}$$

$$\frac{5x-25}{27}$$

slant asymptote
 $y = x + 5$

20

Pre-Calculus: 3.1 – 3.2

Exponential and Logarithmic Functions and Models

Key

Name: _____
 Date: _____ Hour: _____
 SCORE: _____ /60
 Percent Correct: _____ %

Be sure to **SHOW ALL WORK**. Answer questions completely. Be sure to write answers in spaces provided. If work or answers are in another location, please make note of that. There are **60 points** possible.

16/8/4	Correct, complete, with appropriate work or explanations.
12/6/3	Correct strategy, minor errors, appropriate work or explanations.
8/4/2	Starts with appropriate strategy, some understanding, some errors.
4/2/1	Attempted appropriate strategy, minimal understanding.
0	Little or no understanding evident – OR – no work shown.

1. Determine whether each of the functions are exponential growth, decay or not exponential. If it is growth or decay, find the percentage rate for the function. (8 points)

A. $a(x) = 6 \cdot 0.75^x$ Decay, 25%

C. $c(x) = 2 \cdot 4^x$ Growth, 300%

B. $b(x) = -5 \cdot x^3$ Not Exponential

D. $d(x) = x^e$ Not Exponential

2. Choose a basic exponential **growth** function from number 1 and determine the following. (12 points)

Which Function: $2 \cdot 4^x$

A. Base: 4

B. Initial Value: 2

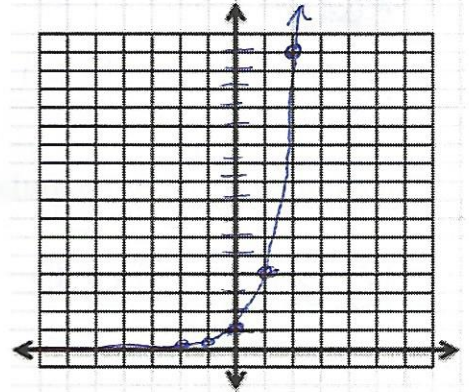
C. Asymptote(s): $y = 0$

D. Domain: $(-\infty, \infty)$

E. Range: $(0, \infty)$

F. Graph:

x	y
-2	1/8
-1	1/2
0	2
1	8
2	32



3. Compute the **exact value** of the function for the given x-value **without** using a calculator. (8 points)

$f(x) = 3 \cdot 5^x$
 $x = 2$

$3 \cdot 5^2 = 3 \cdot 25$

Solution: 75

$g(x) = -4 \cdot 3^x$
 $x = 3/2$

$-4 \cdot 3^{(3/2)}$ $\sqrt{3} \sqrt{9}$
 $-4 \cdot \sqrt{3^3} = -4 \cdot \sqrt{27}$

Solution: $-12\sqrt{3}$

$h(x) = 6 \cdot 2^x$
 $x = -4$

$6 \cdot 2^{-4} = 6 \cdot \frac{1}{16}$
 $\frac{6}{16} = \frac{3}{8}$

Solution: $\frac{3}{8}$

$k(x) = -1 \cdot 4^x$
 $x = -4/2 = -2$

$-1 \cdot 4^{-4/2}$
 $-1 \cdot \frac{1}{16}$ $-1/16$

Solution: $-1/16$

4. Find the exponential function that satisfies the given conditions. (8 points)

A. Initial value = 35, increasing at a rate of 15% per year.

$15\% \rightarrow 0.15 + 1 = 1.15 = b$

Solution: $f(x) = 35(1.15)^x$

B. Initial mass = 15g, decreasing at a rate of 3.7% every 4 days.

$3.7\% \rightarrow 0.037 - 1 = -0.963 = -x$
 $x = 0.963$

Solution: $f(x) = 15(0.963)^{x/4}$

5. The population of Fowlerville is 425,000 and it is decreasing at a rate of 4.2% per year.

a. Write an equation $P(t)$ for the population at time t years from now. (4 points)

$$P(t) = 425,000 \cdot b^x$$

$$r = 4.2\% \rightarrow 0.042 - 1 = -x$$

$$-0.958 = -x \rightarrow x = 0.958 \text{ Solution: } \underline{P(t) = 425,000(0.958)^t}$$

b. Predict the population 5 years from now. (2 points)

$$P(t) = 425,000(0.958)^5$$

$$\text{Solution: } \underline{342,939 \text{ people}}$$

6. The population of Burkeville in 2004 was 5,000. It is currently growing at a rate of 5.5% per year.

A. Write an equation for the exponential situation. (4 points)

$$f(x) = 5000 \cdot b^x$$

$$5.5\% \rightarrow 0.055 + 1 = 1.055$$

$$\text{Solution: } \underline{f(x) = 5000(1.055)^x}$$

B. In what year will the population be double of the original amount? (2 points)

$$10,000 = 5000 \cdot (1.055)^x$$

$$12 \rightarrow \textcircled{13} \text{ 13 years}$$

$$\text{Solution: } \underline{2004 + 13 = \textcircled{2017}}$$

7. Given the following points, determine the exponential equation. (4 points)

(3, 2.048) and (0, 4)

$$a = 4$$

$$f(x) = 4 \cdot b^x$$

$$\sqrt[3]{b^3} = \sqrt[3]{0.512}$$

$$4 \cdot b^3 = 2.048 \quad b = 0.8$$

$$\text{Solution: } \underline{f(x) = 4(0.8)^x}$$

8. Determine exponential equation $f(x)$ and $g(x)$ based on values given in the following table. (8 points)

x	$f(x)$	$g(x)$
-2	8	53.864
-1	4	12.712
0	2	3
1	1	.708
2	.5	.16709

$$f(x)$$

$$a = 2$$

$$2 \cdot b^1 = 1$$

$$b = \frac{1}{2} \text{ or } 0.5$$

$$f(x) = \underline{2 \left(\frac{1}{2}\right)^x}$$

$$\text{or } \underline{2(0.5)^x}$$

$$g(x)$$

$$a = 3$$

$$3 \cdot b^1 = 0.708$$

$$b = 0.236$$

$$g(x) = \underline{3(0.236)^x}$$

Pre-Calculus: 3.1 – 3.2

Exponential and Logarithmic Functions and Models

Name: Key Form B
 Date: _____ Hour: _____
 SCORE: _____ /60
 Percent Correct: _____ %

Be sure to SHOW ALL WORK. Answer questions completely. Be sure to write answers in spaces provided. If work or answers are in another location, please make note of that. There are 60 points possible.

16/8/4	Correct, complete, with appropriate work or explanations.
12/6/3	Correct strategy, minor errors, appropriate work or explanations.
8/4/2	Starts with appropriate strategy, some understanding, some errors.
4/2/1	Attempted appropriate strategy, minimal understanding.
0	Little or no understanding evident – OR – no work shown.

1. Determine whether each of the functions are exponential growth, decay or not exponential. If it is growth or decay, find the percentage rate for the function. (8 points)

A. $a(x) = 4 \cdot 0.25^x$ Decay, 75%
 B. $d(x) = x^e$ Not Exponential

C. $c(x) = 2 \cdot 3^x$ Growth, 200%
 D. $b(x) = -6 \cdot x^4$ Not Exponential

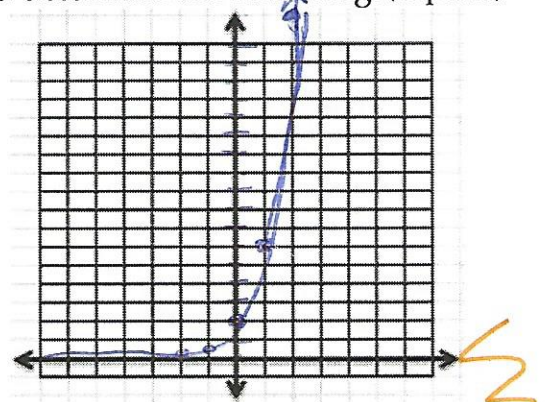
2. Choose a basic exponential **growth** function from number 1 and determine the following. (12 points)

Which Function: $f(x) = 2 \cdot 3^x$

- A. Base: 3
 B. Initial Value: 2
 C. Asymptote(s): $y = 0$
 D. Domain: $(-\infty, \infty)$
 E. Range: $(0, \infty)$

F. Graph:

x	y
-2	$\frac{2}{9}$
-1	$\frac{2}{3}$
0	2
1	6
2	18



3. Compute the **exact value** of the function for the given x-value **without** using a calculator. (8 points)

$f(x) = -5 \cdot 3^x$
 $x = 2$

$-5 \cdot (3)^2$
 $-5 \cdot 9$

Solution: -45

$g(x) = 4 \cdot 2^x$
 $x = 3/2$

$4 \cdot \sqrt{2^3}$
 $4 \cdot \sqrt{8} = 4 \cdot 2\sqrt{2}$

Solution: $8\sqrt{2}$

$h(x) = 6 \cdot 3^x$
 $x = -3$

$6 \cdot 3^{-3}$
 $6 \cdot \frac{1}{27} = \frac{2}{9}$

Solution: $\frac{2}{9}$

$k(x) = -1 \cdot 3^x$
 $x = -6/2 = -3$

$-1 \cdot 3^{-3}$
 $= -1 \cdot \frac{1}{27}$

Solution: $-\frac{1}{27}$

4. Find the exponential function that satisfies the given conditions. (8 points)

A. Initial value = 27, increasing at a rate of 30% per year.

$\frac{30\%}{100} = 0.30 + 1 = 1.30$

Solution: $f(x) = 27(1.30)^x$

B. Initial mass = 79g, decreasing at a rate of 2.8% every 3 days.

$2.8\% \rightarrow 0.028 - 1 = -0.972 = -x$

$x = 0.972$

Solution: $f(x) = 79(0.972)^{x/3}$

5. The population of Fowlerville is 375,000 and it is decreasing at a rate of 5.6% per year.

a. Write an equation $P(t)$ for the population at time t years from now. (4 points)

$$5.6\% \rightarrow 0.056$$

$$1 - 0.056 = 0.944$$

$$\text{Solution: } P(t) = 375,000(0.944)^t$$

b. Predict the population 5 years from now. (2 points)

$$P(t) = 375,000(0.944)^t \quad t=5$$

$$= 375,000(0.944)^5$$

$$\text{Solution: } \underline{281,120 \text{ people}}$$

6. The population of Burkeville in 2001 was 6,500. It is currently growing at a rate of 4.7% per year.

A. Write an equation for the exponential situation. (4 points)

$$f(x) = 6,500(1.047)^x$$

$$4.7\% \rightarrow 0.047 + 1 = 1.047$$

$$\text{Solution: } \underline{f(x) = 6500(1.047)^x}$$

B. In what year will the population be double of the original amount? (2 points)

$$6500 \times 2 = \text{13,000} \quad 15 \rightarrow 16$$

$$13,000 = 6500(1.047)^x$$

$$2 = 1.047^x$$

$$15.1 \text{ years} \approx 15 \text{ years}$$

$$\text{Solution: } \underline{2001 + 15 = 2016}$$

7. Given the following points, determine the exponential equation. (4 points)

(3, 1.715) and (0, 5)

$$f(x) = 5 \cdot b^x$$

$$5 \cdot b^3 = 1.715 \rightarrow b^3 = 0.343$$

$$b = 0.7$$

$$\text{Solution: } f(x) = \underline{5(0.7)^x}$$

8. Determine exponential equation $f(x)$ and $g(x)$ based on values given in the following table. (8 points)

x	$f(x)$	$g(x)$
-2	5.2126	.1875
-1	4.5662	.75
0	4	3
1	3.504	12
2	2.6889	48

$$a = 4 \quad f(x)$$

$$4 \cdot b^1 = 3.504$$

$$b = 0.876$$

$$f(x) = \underline{4(0.876)^x}$$

$$a = 3 \quad g(x)$$

$$3 \cdot b^1 = 12$$

$$b = 4$$

$$g(x) = \underline{3 \cdot 4^x}$$