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Date: 10/9/14 Hour: All

1.3 - 1.5 Test Review:

1. Find $(f + g)(x)$, $(f - g)(x)$, $(f * g)(x)$, and $(\frac{f}{g})(x)$ given that $f(x) = x^2 + 3$ and $g(x) = 2x + 4$.

$$\begin{aligned} &(f+g)(x) \\ &f(x) + g(x) \\ &x^2 + 3 + 2x + 4 \\ &\quad \xrightarrow{\hspace{2cm}} \\ &\quad \underline{x^2 + 2x + 7} \\ &\text{(can't be factored)} \end{aligned}$$

$$\begin{aligned} &(f-g)(x) \\ &x^2 + 3 - (2x + 4) \\ &x^2 + 3 - 2x - 4 \\ &\quad \xrightarrow{\hspace{2cm}} \\ &\quad \underline{x^2 - 2x - 1} \\ &\text{(can't be factored)} \end{aligned}$$

$$\begin{aligned} &(f * g)(x) \\ &f(x) * g(x) \\ &(x^2 + 3)(2x + 4) \\ &\quad \xrightarrow{\hspace{2cm}} \\ &\quad \underline{2x^3 + 4x^2 + 6x + 12} \end{aligned}$$

$$\begin{aligned} &(\frac{f}{g})(x) \\ &\frac{f(x)}{g(x)} \\ &\frac{x^2 + 3}{2x + 4} \\ &\quad \xrightarrow{\hspace{2cm}} \\ &\quad \text{(can't be simplified)} \end{aligned}$$

a) $(f + g)(x) = \underline{x^2 + 2x + 7}$

b) $(f - g)(x) = \underline{x^2 - 2x - 1}$

c) $(f * g)(x) = \underline{\frac{2x^3 + 4x^2 + 6x + 12}{x^2 + 3}}$

d) $(\frac{f}{g})(x) = \underline{\frac{x^2 + 3}{2x + 4}}$

2. Find $(f + g)(2)$, $(f - g)(-1)$, $(f * g)(4)$, and $(\frac{f}{g})(3)$ given that $f(x) = 2 + 3x - x^2$ and $g(x) = \sqrt{x + 1}$.

$$\begin{aligned} &(f+g)(2) \\ &2 + 3(2) - (2)^2 + \sqrt{2+1} \\ &2 + 6 - 4 + \sqrt{3} \\ &\quad \xrightarrow{\hspace{2cm}} \\ &\quad \underline{4 + \sqrt{3}} \end{aligned}$$

$$\begin{aligned} &(f-g)(-1) \\ &2 + 3(-1) - (-1)^2 - \sqrt{-1+1} \\ &2 - 3 - 1 - \sqrt{0} \\ &2 - 3 - 1 \\ &\quad \xrightarrow{\hspace{2cm}} \\ &\quad \underline{-2} \end{aligned}$$

$$\begin{aligned} &(f * g)(4) \\ &(2 + 3(4) - (4)^2)(\sqrt{4+1}) \\ &(2 + 12 - 16)(\sqrt{5}) \\ &\quad \xrightarrow{\hspace{2cm}} \\ &\quad \underline{-2\sqrt{5}} \end{aligned}$$

$$\begin{aligned} &(\frac{f}{g})(3) \\ &\frac{2 + 3(3) - (3)^2}{\sqrt{3+1}} \\ &\frac{2 + 9 - 9}{\sqrt{4}} \\ &\frac{2}{2} = \underline{1} \end{aligned}$$

a) $(f + g)(x) = \underline{4 + \sqrt{3}}$

b) $(f - g)(x) = \underline{-2}$

c) $(f * g)(x) = \underline{-2\sqrt{5}}$

d) $(\frac{f}{g})(x) = \underline{1}$

$$f(x) = x^2 + 3$$

$$g(x) = 2x + 4$$

$$g(f(x)) = g(x^2 + 3)$$

$$= 2(x^2 + 3) + 4$$

$$2x^2 + 6 + 4 = 2x^2 + 10$$

3. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ for #1 and #2.

1. $f(g(x)) = f(2x + 4)$

$$= (2x + 4)^2 + 3$$

$$= (2x + 4)(2x + 4) + 3$$

$$= 4x^2 + 8x + 8x + 16 + 3$$

$$4x^2 + 16x + 19$$

a) $(f \circ g)(x) = 4x^2 + 16x + 19$

b) $(g \circ f)(x) = 2x^2 + 10$

2. $f(g(x)) = f(\sqrt{x+1})$

$$2 + 3\sqrt{x+1} - (\sqrt{x+1})^2$$

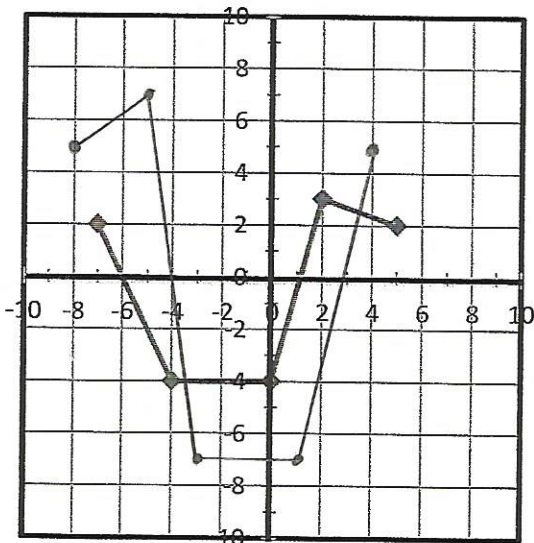
$$2 + 3\sqrt{x+1} - (x+1) = 2 + 3\sqrt{x+1} - x - 1 = 3\sqrt{x+1} - x + 1$$

$g(f(x)) = g(2 + 3x - x^2)$

c) $(f \circ g)(x) = 3\sqrt{x+1} - x + 1$

d) $(g \circ f)(x) = \sqrt{-x^2 + 3x + 3}$

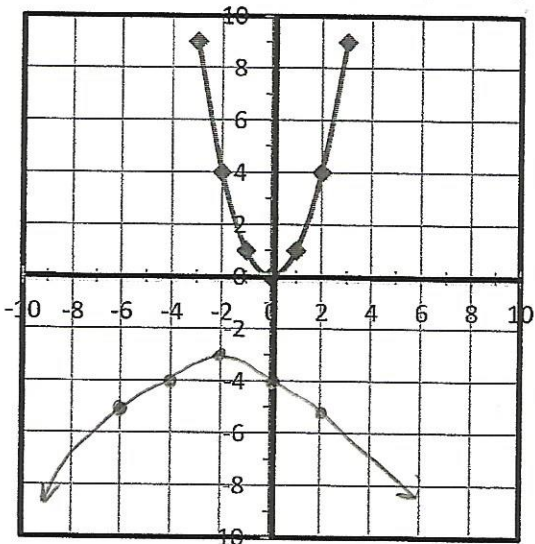
4. Perform the appropriate graph transformations and draw the new graph.



$$y = 2f(-x - 3) + 1$$

x	y	$\xrightarrow{x+3}$	x	y	$\xrightarrow{2 \cdot y}$	x	y	$\xrightarrow{-1 \cdot x}$	x	y	$\xrightarrow{y+1}$	x	y
-7	2		-4	2		-4	4		4	4		4	5
-4	-4		-1	-4		-1	-8		1	-8		1	-7
0	-4		3	-4		3	-8		-3	-8		-3	-7
2	3		5	3		5	6		-5	6		-5	7
5	2		8	2		8	4		-8	4		-8	5

5. Perform the appropriate graph transformations and draw the new graph.



$$f(x) = -\left(\frac{1}{2}x + 1\right)^2 - 3$$

Parent Function: $f(x) = x^2$

x	y	$\xrightarrow{x-1}$	x	y	$\xrightarrow{2 \cdot x}$	x	y	$\xrightarrow{-1 \cdot y}$	x	y	$\xrightarrow{y-3}$	x	y
-2	4		-3	4		-6	4		-6	-4		-6	-7
-1	1		-2	1		-4	1		-4	-1		-4	-4
0	0		-1	0		-2	0		-2	0		-2	-3
1	1		0	1		0	1		0	-1		0	-4
2	4		1	4		2	4		2	-4		2	-7

6. Find the inverse of the function: $f(x) = \frac{2x-1}{x+3}$.

$$y = \frac{2x-1}{x+3}$$

$$x = \frac{2y-1}{y+3}$$

$$x(y+3) = 2y-1$$

$$xy + 3x = 2y - 1$$

$$3x + 1 = 2y - xy$$

$$3x + 1 = y(2-x)$$

$$y = \frac{3x+1}{2-x}$$

$$f^{-1}(x) = \frac{3x+1}{2-x}$$

7. State the transformations occurring in each function below:

1. $f(x) = -\frac{1}{3}\sqrt{x-2} + 1$

1. H = H. shift right by 2
2. S = V. shrink by 1/3
3. R = Reflection over x-axis
4. V = V. shift up by 1

2. $f(x) = |-4x + 5| - 2$

1. H = H. shift left by 5
2. S = H. shrink by 1/4
3. R = Reflection over y-axis
4. V = V. shift down by 2

8. Are f and g inverses of each other? Prove it using composition of functions.

$f(x) = x^2 + 1$ and $g(x) = \sqrt{x-3}$

$$f(g(x)) = f(\sqrt{x-3})$$

$$= (\sqrt{x-3})^2 + 1 = x - 3 + 1 = \boxed{x-2}$$

$$g(f(x)) = g(x^2 + 1)$$

$$= \sqrt{(x^2 + 1) - 3}$$

$$= \sqrt{x^2 - 2}$$

$f(x)$ and $g(x)$ are not inverses.

9. Are h and k inverses of each other? Prove it using composition of function.

$h(x) = 3x - 1$ and $k(x) = \frac{1}{3}x + \frac{1}{3}$

$$h(k(x)) = h\left(\frac{1}{3}x + \frac{1}{3}\right)$$

$$= 3\left(\frac{1}{3}x + \frac{1}{3}\right) - 1$$

$$= x + 1 - 1 = \boxed{x}$$

$$k(h(x)) = k(3x - 1)$$

$$= \frac{1}{3}(3x - 1) + \frac{1}{3} \rightarrow x - \frac{1}{3} + \frac{1}{3}$$

$$= \boxed{x}$$

$h(x)$ and $k(x)$ are inverses

If you have extra time: What are the ten parent functions?

1. $f(x) = x$

2. $f(x) = x^2$

3. $f(x) = x^3$

4. $f(x) = \sqrt{x}$

5. $f(x) = \frac{1}{x}$

6. $f(x) = |x|$

7. $f(x) = e^x$

8. $f(x) = \ln(x)$

9. $f(x) = \sin(x)$

10. $f(x) = \cos(x)$