# 2.3 – 2.4 Test Review

Name: ______________________
Date: _______________ Hour: ___

## End Behavior, Zeros, Extrema, Multiplicity and Division

1. **End Behavior:**

| Degree | Leading Coefficient | $x \to -\infty$ | $x \to +\infty$ |
|---|---|---|---|
| Even | Positive | $f(x) \to \infty$ | $f(x) \to \infty$ |
| Even | Negative | $f(x) \to -\infty$ | $f(x) \to -\infty$ |
| Odd | Positive | $f(x) \to -\infty$ | $f(x) \to \infty$ |
| Odd | Negative | $f(x) \to \infty$ | $f(x) \to -\infty$ |

A. State the end behavior for the following functions:

i. $f(x) = 3x^4 - 2x^2 + 1$ — $\infty$ and $\infty$

ii. $f(x) = -1x^3 + 5x^2 + 2x - 7$ — $\infty$ and $-\infty$

iii. $f(x) = 4x^5 - 3x + 6$ — $-\infty$ and $\infty$

2. **Zeros, Extrema and Multiplicity:**

A. Where are zeros found on a graph?

Zeros are located at any point where the graph crosses the x-axis or when $y = 0$.

B. What are extrema?

Extrema are maximum or minimum points of the graph.

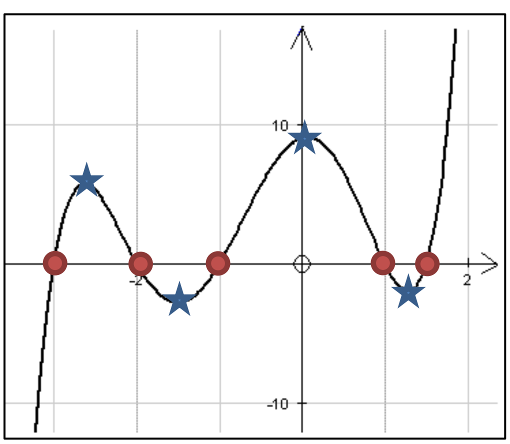
C. What is multiplicity?

Multiplicity is the number of times a zero occurs in a polynomial function.

i. **Even:** The graph kisses the x-axis.

ii. **Odd:** The graph crosses the x-axis.

D. Given a graph, state the following:




i. Number of Zeros: 5

ii. Number of Extrema: 4

E. State the degree and zeros of the polynomial function. State the multiplicity of each zero and what the behavior of the graph is at that zero (crosses/kisses).

$$f(x) = x^2(x-3)^4(x+1)^{1}$$

Degree: 7

| Zeros | Multiplicity | Crosses/Kisses |
|---|---|---|
| $x = 0$ | 2 | Kisses |
| $x = 3$ | 4 | Kisses |
| $x = -1$ | 1 | Crosses |

3. **Long Division:** Divide $f(x)$ by $d(x)$.

$$f(x) = x^3 - 5x^2 + 3x - 15 \quad \text{and} \quad d(x) = x^2 + 3$$

$$\begin{array}{r} \mathbf{x - 5} \\ \mathbf{x^2 + 0x + 3}\ \big)\ \overline{\mathbf{x^3 - 5x^2 + 3x - 15}} \\ \underline{-\ (\mathbf{x^3 + 0x^2 + 3x})} \\ \mathbf{0 - 5x^2 + 0x - 15} \\ \underline{-\ (\mathbf{-5x^2 + 0x - 15})} \\ \mathbf{0} \end{array}$$

**PF:** $(x^2+3)(x-5)+0$

$= d(x) * q(x) + r(x)$

**FF:** $(x-5) + \frac{0}{(x^2+3)}$

$= q(x) + \frac{r(x)}{d(x)}$

4. Without dividing, is $(x-6)$ a factor of $x^3 - 6x^2 + 5x - 2$? **(Show what you did!!)**

**Use the Factor Theorem:** $f(k) = 0$ when $x - k$ is the divisor. In other words, set $x - k = 0$ and solve for the value of k. Take that value and plug it in for $x$ to see if you will get a result of 0.

$x - k = 0 \rightarrow x - 6 = 0 \rightarrow x = 6$ which means $k = 6$.

$$f(6) = (6)^3 - 6(6)^2 + 5(6) - 2 = 28 \neq 0$$

Thus, $x - 6$ is **not** a factor.

5. What can the remainder theorem be used for? ~~(Use it on #3).~~

The **remainder theorem** can be used as a check on your long and synthetic division. The remainder theorem states that when you have a **linear** divisor of $x - k$, you can do $f(k)$ and if that equals the remainder you found in your division than you have checked you are correct.

6. **Synthetic Division:** Divide $3x^3 - 2x^2 + 3x - 4$ by $x - 3$

| 3 | 3 | −2 | 3 | −4 |
|---|---|---|---|---|
| + | ↓ | 9 | 21 | 72 |
| | 3 | 7 | 24 | 68 |

$\mathbf{3x^2 + 7x + 24} \qquad \mathbf{R: 68}$

$\mathbf{x - 3 = 0 \rightarrow x = 3}$

**Final Equation:**

$\mathbf{(3x^2 + 7x + 24)(x - 3) + 68}$

7.

8. **Rational Zero Theorem:** Find all of the real zeros for the polynomial function:

$$f(x) = 2x^4 + 4x^3 - 41x^2 - 31x + 12$$

$$\frac{p(factors\ of\ 12)}{q(factors\ of\ 2)} = \frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1, \pm2} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{3}{2}$$

After plugging my equation into the calculator and looking at the table, I see that I have two zeros: $-1$ *and* $4$. So I do synthetic division with one of them.

| −1 | 2 | 4 | −41 | −31 | 12 |
|---|---|---|---|---|---|
| + | ↓ | −2 | −2 | 43 | −12 |
| | 2 | 2 | −43 | 12 | 0 |

| 4 | 2 | 2 | −43 | 12 |
|---|---|---|---|---|
| + | ↓ | 8 | 40 | 12 |
| | 2 | 10 | −3 | 0 |

$2x^2 + 10x - 3$

**Factor:** $2x^2 + 10x - 3$ using the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-10 \pm \sqrt{(10)^2 - 4(2)(-3)}}{2(2)}$$

$$\rightarrow \frac{-10 \pm \sqrt{100+24}}{4} \rightarrow \frac{-10 \pm \sqrt{124}}{4}$$

$$\rightarrow \frac{-10 \pm 2\sqrt{31}}{4} \rightarrow \frac{-5 \pm \sqrt{31}}{2}$$

$\sqrt{124} = \sqrt{4} * \sqrt{31} = 2\sqrt{31}$

All three numbers are divisible by 2.

- Thus, your zeros are: $-1, 4, \frac{-5+\sqrt{31}}{2}, \frac{-5-\sqrt{31}}{2}$.

9. Extra Practice:
   1. Long and Synthetic Division:
      A. $f(x) = 2x^4 - 3x^3 + 9x^2 - 14x + 7$ and $d(x) = x^2 + 4$
         I. $(x^2 + 4)(2x^2 - 3x + 1) + (-2x + 3)$
         II. $\mathbf{-2x + 3}$ is the remainder.
      B. $f(x) = x^4 + 3x^3 + x^2 - 3x + 3$ and $d(x) = x + 2$
         I. $(x + 2)(x^3 + x^2 - x - 1) + 5$
      C. $f(x) = 2x^3 - 7x^2 + 4x - 5$ and $d(x) = x - 3$
         I. $(x - 3)(2x^2 - x + 1) - 2$

**If you need help figuring any of these questions out, you can email me.**
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